New Meridian

MasteryGuide™ Mathematics

Assessment Specifications and Blueprints

February 2024, v1.0

New Meridian

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Mathematics Design

System Overview

The mathematics through-year assessment system is organized around 12 strands of mathematical understanding that represent the content of the grade, as defined in the Common Core State Standards for Mathematics (CCSS-M). Each strand encompasses a small set of related content standards. The strands were developed with reference to foundational documents such as Student Achievement Partners' *Coherence Map* (Achieve the Core, 2012) and the *Progressions for the Common Core State Standards in Mathematics* (Common Core Standards Writing Team, 2023), as well as common high-quality curricula (i.e., Illustrative Mathematics, Eureka Math, EngageNY).

Across the year, students take 12 short tests, termed "testlets," to assess their mastery of the content of the strands. Each testlet assesses student proficiency in one strand. The grain size of the strands ensures that the sequence of testlet administration can be configured to meet the local scope and sequence of the classroom, school, or district participating.

Thus, students will only take any testlet after they have had the opportunity to learn the content of that strand.

Item and Form Development

Each testlet is built following a Cognitive Diagnostic Modeling q-matrix design, with two latent attributes to be observed. Some attributes correspond one-to-one with a CCSS-M standard; some attributes align to part of a single standard, while others encompass skills from multiple standards. In some strands, the attributes represent a rearrangement of the knowledge, skills, and abilities described by multiple standards. Each item-interaction is designed to measure one attribute derived from the CCSS-M.

Testlets are designed to gather evidence of student development along a progression of learning within and across grades, informed by the *Progressions for the Common Core State Standards for Mathematics* (Common Core Standards Writing Team, 2023). In each strand, items are written at several levels of complexity that reflect a variety of depths of understanding of content. We are investigating and aligning the items to several frameworks for cognitive complexity, including Webb's Depth of Knowledge (Webb, 1997; 2006), Aspects of Rigor (Achieve, 2019), and New Meridian's Cognitive Complexity Framework for Mathematics to ensure the items provide coverage of a range of skills across a range of complexity categories.

The items within each testlet also reflect a range of difficulty levels. During the design phase of the development cycle, specialists review multiple sources of information to create descriptions of easy, medium, and difficult items for each strand. These levels are defined by the percentage of students expected to provide a full-credit response (the item's p-value); specialists tag items based on a calibrated understanding of how the unique aspects of each item may contribute to the actual p-value realized upon item analysis.

Items are tagged with short descriptions of the stimulus (e.g., real-world context, analog clock, area model) as well as common errors and misconceptions (e.g., student reverses the left-to-right, ten-to-one relationship in adjacent place values) to support alignment with the intended attributes and enhance future reporting efforts. During item authoring and review, subject matter experts identify possible misconceptions that students may hold. When possible, distractors on selected-response items and anticipated incorrect responses in constructed-

response items are tagged with these misconceptions. Upon review of actual student responses post-administration, the list will be revised as needed.

For the SY24/25 school year, all item-interactions are machine-scorable and scored dichotomously (i.e., 0 or 1). Interaction types include Choice (single- and multiple-select), Inline Choice, Gap Match, Graphic Gap Match, Graphic Hotspot, Matching Table Grid, Order, and Text Entry (short numeric response only).

Items are developed by New Meridian's team of content, accessibility and fairness, and editorial experts. Representatives from the Montana Office of Public Instruction (OPI) and the Louisiana Department of Education (LDOE) review the items, followed by a review conducted by a cadre of educators recruited by the states.

Items are authored in New Meridian's item bank management system. The items are ingested to ATS' delivery platform where they undergo a forms UAT review by New Meridian's content team and representatives from OPI and LDOE. Accommodated forms include paper-pencil delivery, Spanish transadapted for online delivery, Spanish transadapted for paper-pencil delivery, and translation into American Sign Language (ASL) via embedded videos for online delivery.

Testlet Specifications

For the SY24/25 School year, all testlets adhere to the following specifications:

Overview	
Number of strands assessed per testlet	1
Number of reporting attributes per strand	2
Number of reporting attributes assessed per testlet	2
Number of items per testlet	9-13
Distribution	
Score points per item	All items scored dichotomously (i.e., 0 or 1)

Interaction types	Approximately: 40% choice (single- or multiple-select multiple choice) 40% constructed response (numeric text entry only) 20% technology-enhanced (matching table, inline choice, gap match, hot spot, hot text, order)
Reporting Attributes	2 reporting attributes per testlet Approximately 45–55% for each attribute Items must assess the reporting attribute and align to a CCSS standard.
Complexity (as measured using Webb's Depth of Knowledge)	Approximately: 60–75% DOK 1 25–40% DOK 2
Expected difficulty	Approximately: 20–30% easy (expected p > 0.7) 40–60% medium (0.35 < expected p < 0.7) 10–20% hard (expected p < 0.35)

Grade 3

General Discussion

Grade 3 testlets place an emphasis on the critical work of third grade as students:

- Develop an understanding of and familiarity with the concept and application of multiplication and division as representing the combining and partitioning of equal groups of objects.
- Develop an understanding of area as a two-dimensional measurement attribute and its relationship to multiplication and division.
- Develop an understanding of fractions as numbers.

In third grade, students build on their foundational understanding of repeated addition using arrays as they explore and progress toward fluency in multiplication. They come to understand multiplication as the combining of equal groups, where the product represents the total number of objects or units when the groups are combined. Students recognize the relationship between multiplication and division, where division can represent either repeated subtraction (quotative) or equal partitioning/sharing (partitive). Strands 1–3 focus exclusively on

multiplication and division, while strands 4, 5, 10, and 11 provide students with opportunities to use those skills in other contexts.

Third grade students develop their understanding of multiplication while learning concepts of area. They recognize area as a two-dimensional measurement that is reported in the number of square units, in contrast with previous measurement work with one-dimensional, linear units. Working with area also gives students the context in which to practice multiplication skills and the properties of operations, both within 10 × 10 and with multiples of 10. Strands 9–10 focus specifically on area concepts and applications.

Finally, in third grade, students formalize their understanding of fractions. In previous grades, students began to see fractions as representing partitions of a geometric model. In third grade, students develop an understanding that fractions are numbers that can be found on a number line. They extend previous understanding of length models to understand that a fraction is located at the end of the interval which starts at 0 and has a length of that fraction. They recognize equivalent fractions as those that are located at the same point on a number line, including seeing that a fraction can be equivalent to a whole number. Strands 7–8 focus on initial fraction concepts, while strand 11 offers an opportunity to apply fraction understanding in the context of data and measurement.

Students taking grade 3 testlets do not have access to a calculator or reference sheet.

Strands

Strand 1: Concepts of Multiplication and Division

Working with numbers ranging from 0 to 10, students demonstrate an understanding of the meaning of multiplication and division. They identify and interpret products and quotients in real-world contexts, and they use multiplication and division skills to solve one-step real-world problems.

Attributes

- The student interprets products and quotients of whole numbers.
- The student solves one-step real-world problems by multiplying or dividing within 10×10.

CCSS-M

- 3.OA.A.1, 3.OA.A.2, 3.OA.A.3

Additional Specifications

- Students may be asked to interpret products or quotients within 10×10.
- Students will only be expected to perform calculations with factors of 0, 1, 2, 3, 4, and 5.
- Some items will include a visual model to represent the calculation.
- Division items may include partitive or quotative division situations.
- Items will be evenly distributed across situations involving groups, arrays, and measurement quantities.

Strand 2: Multiplication and Division Equations

Students demonstrate an understanding of the inverse relationship between multiplication and division by finding an unknown number in a multiplication or division equation, including representing an equation division as a multiplication equation with an unknown factor.

Attributes

- The student determines an unknown number in a multiplication or division equation relating three whole numbers, within 10×10.
- The student understands division as an unknown-factor problem.

CCSS-M

- 3.OA.A.4, 3.OA.B.6

Additional Specifications

- Items will represent an even distribution of multiplication and division.
- Items will represent an even distribution of the properties of multiplication and division.
- Based on the intended difficulty, the unknown number in an equation may be placed in any location.

Strand 3: Multiply and Divide Within 100

Students demonstrate the use of the associative, commutative, and distributive properties to multiply and divide within 10×10. They also complete multiplication and division facts.

Attributes

- The student understands and uses properties of operations to multiply and divide.
- The student fluently multiplies and divides within 10×10.

CCSS-M

- 3.OA.B.5, 3.OA.C.7

Additional Specifications

- Items will be equally distributed across the 'quadrants' of the 10×10 multiplication table:

    - both factors 5 or less
    - one factor 5 or less and the other factor between 6 and 10 inclusive
    - both factors between 6 and 10 inclusive
- Items will represent an even distribution of the properties of multiplication and division.

Strand 4: Time, Liquid Volume, and Mass

Students demonstrate mastery of reading time from an analog clock, determining elapsed time less than one hour, and solving real-world problems involving intervals of time. They use measurement scales based on the number line to estimate and find the liquid volume or mass of an object and solve one-step real-world problems regarding these measurements.

Attributes

- The student tells time and solves problems involving intervals of time.
- The student solves real-world problems involving liquid volume and mass.

CCSS-M

- 3.MD.A.1, 3.MD.A.2

Additional Specifications

- Calculations are restricted to the same ones described in 3.OA.C.7, 3.NBT.A.2, and 3.NBT.A.3.
- Students may be asked to apply estimation and/or reasoning skills as described in 3.OA.D.8.
- Real-world problems in this strand are intended to provide a context for students to apply newly developing multiplication and division skills. Most items will include multiplication or division, rather than addition or subtraction.
- When models are presented in the stimulus, they will be numerical and partitioned proportionally, such as lines marking liquid volume on a bucket, an analog clock, an analog dial measurement scale, or a typical number line.
- In items about elapsed time, the duration will be less than one hour. The duration may or may not cross the hour.

Strand 5: Real-World Problems and Patterns

Students demonstrate a growing capacity to solve two-step real-world problems, including using rounding and other reasoned estimation strategies. They represent the problems with situation and solution equations, representing an unknown with a letter. Students also use their understanding of the properties of operations to explain arithmetic patterns associated with multiplication.

Attributes

- The student represents and solves two-step real-world problems.
- The student identifies arithmetic patterns and explains them using the properties of operations.

CCSS-M

- 3.OA.D.8, 3.OA.D.9

Additional Specifications

- Items aligned to 3.OA.D.8 will involve at least one multiplication or division step in alignment with the grade-level emphasis on those operations.
- Standard 3.NBT.A.1 will only be assessed as a part of solving real-world problems. No item will be aligned to 3.NBT.A.1 in isolation.
- Items aligned to 3.OA.D.9 will focus on arithmetic patterns related to multiplication or found in a multiplication table, not addition. Students will not be asked about geometric patterns.
- Real-world problems may involve equations or expressions that use letters to represent an unknown number. The letter selected will be logically drawn from the object it represents, e.g., the number of pennies would be represented by the letter “p” while the number of nickels would be represented by the letter “n.”

Strand 6: Place Value and Operations in Base Ten

Students demonstrate fluency with addition and subtraction of whole numbers within 1,000, including demonstrating an understanding of strategies based on place value. They apply place value understanding regarding tens and ones as well as the associative, commutative, and distributive properties of multiplication and multiply with larger whole numbers, specifically multiplying a two-digit multiple of 10 by a single-digit number.

Attributes

- The student fluently adds and subtracts within 1,000.
- The student multiplies two-digit multiples of 10 by one-digit whole numbers.

CCSS-M

- 3.NBT.A.2, 3.NBT.A.3

Additional Specifications

- Items aligned to 3.NBT.A.2 may present the addition or subtraction vertically or horizontally.
- In alignment with the progression of learning in addition and subtraction, most items aligned to 3.NBT.A.2 will require at least one regrouping of place values.

- In alignment with the progression of learning in addition and subtraction, most items in this strand that align to 3.NBT.A.2 will have three-digit numbers for all three values (addends and sum, or minuend, subtrahend, and difference). No items will include one-digit numbers or sums/minuends less than 100.
- In items aligned to 3.NBT.A.3, the 'quadrants' of the 10×10 multiplication table will be represented evenly among both the one-digit and the two-digit multiple of ten factors:
  - a number in the range 1–5 times a number in the range 10–50
  - a number in the range 1–5 times a number in the range 60–90
  - a number in the range 6–9 times a number in the range 10–50
  - a number in the range 6–9 times a number in the range 60–90
- Most items in the strand will be pure math (no real-world context). In items that do include a real-world context, the goal is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.

Strand 7: Understand Fractions as Numbers

In previous grades, students worked with shapes equally partitioned into two, three, or four parts. In grade 3, students come to understand that fractions are numbers that can be located along a number line, some between whole numbers and others located at the same point as a whole number. They understand that a unit fraction represents one part of a whole when the whole is partitioned into equal parts, and it is located at the end of the first subdivision when the interval from 0 to 1 on the number line is equally subdivided. They apply this knowledge to describe the size and location on a number line of a non-unit fraction.

Attributes

- The student represents unit fractions using area and length models.
- The student extends unit fraction understanding to represent non-unit fractions.

CCSS-M

- 3.NF.A.1, 3.NF.A.2, 3.NF.A.2.a, 3.NF.A.2.b, 3.G.A.2

Additional Specifications

- Items not assessing unit fractions will be evenly distributed across fractions less than 1 and fractions greater than 1 (represented in $\frac{a}{b}$ format).
- In alignment with the grade-level understanding of a fraction as a number, more items will use number line models than area models.
- Some items in the strand will include a real-world context. The goal of this is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.

Strand 8: Compare and Find Equivalent Fractions

In previous grades, students measured an object using two different length units and recognized that the count of length units is inversely related to the size of the length unit. In grade 3, this understanding serves as a foundation for comparing unit fractions (i.e., length units on a number line model). Students also compare fractions with the same denominator, leveraging their understanding that a fraction represents a count of parts of a given size. They also understand that the two fractions must refer to the same whole to generate a valid comparison.

Attributes

- The student recognizes and generates equivalent fractions and explains their equivalency using visual models.
- The student compares fractions and justifies the comparison using visual models.

CCSS-M

- 3.NF.A.3, 3.NF.A.3.a, 3.NF.A.3.b, 3.NF.A.3.c, 3.NF.A.3.d

Additional Specifications

- Fractions are limited to denominators of 1, 2, 3, 4, 6, and 8. Incorrect answer options may include other denominators as indicated by the needs of the individual item.
- In alignment with an understanding of fractions as numbers, more items in this strand will use number line models than area models.
- Some items in the strand will include a real-world context. The goal of this is to provide the student a relatable entry point to the mathematics being assessed, not to assess problem-solving.

Strand 9: Unit Squares and Square Units

Students extend their understanding of measurement to include two-dimensional units, recognizing a unit square as the measurement unit used to find the number of units that cover a figure. They understand that unit squares must completely cover the shape, without overlaps or gaps between, for their number to represent the area, analogous to how linear units must be aligned exactly end-to-end, including units found on measurement scales and intervals on the number line. Students count the number of unit squares to find a rectangle's area and relate this to multiplication, first finding the area of an array of unit squares by multiplying the number of unit squares in each row (or column) by the number of rows (or columns), then multiplying the number of rows by the number of columns, and finally, recognizing the number of rows and columns as equivalent to the side lengths, by multiplying the measure of each of the two dimensions of the rectangle.

Attributes

- The student understands concepts of area.
- The student demonstrates the area of a rectangle can be found by counting unit squares and by multiplying side lengths.

CCSS-M

- 3.MD.C.5, 3.MD.C.5.a, 3.MD.C.5.b, 3.MD.C.6, 3.MD.C.7, 3.MD.C.7.a

Additional Specifications

- Items in this strand may include improvised units for area measurement (e.g., square crackers used as unit squares).
- Most items will involve multiplication of the number of units in a row by the number of rows (or columns) or of the side lengths, rather than counting unit squares.
- Multiplication is restricted to general grade-level limitations.

Strand 10: Solve Area Problems

Students multiply side lengths to find the area of rectangular figures in mathematical and real-world contexts. They recognize that area is additive; when two rectangles are joined, the area of the whole figure is equal to the sum of the areas of the two rectangles. Students use this understanding to find the area of composite rectilinear figures and to demonstrate and explain the distributive property of multiplication over addition, writing expressions and equations that model the process of finding the area.

Attributes

- The student solves real-world and mathematical problems involving area.
- The student uses area models to represent and explain the distributive property of multiplication over addition.

CCSS-M

- 3.MD.C.7, 3.MD.C.7.b, 3.MD.C.7.c, 3.MD.C.7.d

Additional Specifications

- Measurement units are limited to centimeters, inches, feet, and generic “units.”
- Some items assessing the distributive property of multiplication over addition, or the area of composite rectilinear figures, will include a rectangle with a side length between 11 and 19 units (i.e., $10 + n$), inclusive, where the length is decomposed into 10 units plus *n* units.
- The strand will include items that assess finding the area of a rectilinear figure composed of two non-overlapping rectangles.
- Items will reference the “side lengths” of the rectangle, not the “length and width.”

- If necessary, parentheses should only be used to group the addends of a decomposed side length.

Strand 11: Data and Graphing

Students use multiplication skills while working with scaled categorical data displays such as scaled bar graphs and scaled pictographs. They build on skills that were developed in previous years to create and analyze more sophisticated data displays, solving one- and two-step problems based on data they retrieve from the graph. Students extend their previous understanding of length units to include fractional measurements; line plots give students a context to demonstrate their understanding of fractions as numbers that can be located on a number line.

Attributes

- The student represents categorical data on scaled graphs and solves real-world problems using data presented on scaled graphs.
- The student measures lengths and represents the measurement data on a line plot.

CCSS-M

- 3.MD.B.3, 3.MD.B.4

Additional Specifications

- All categorical data displays will use scales of 2, 3, 4, 5, or 10. Bars in a bar graph may stop at the scale line or halfway in between. Pictographs may use half-icons only if the icon does not represent a living thing nor is the icon itself a representation of a living thing.
- Items may include images of “broken” rulers (e.g., a jagged break line appears at the left end of the ruler such that the first mark shown and labeled is the 2-inch mark).
- All lengths will be measured to the nearest half or fourth of an inch. Lengths may be reported and plotted using a mix of denominators and/or whole numbers. Corresponding line plots may have all or some tick marks labeled.

Strand 12: Two-Dimensional Geometric Figures

Students understand that individual shapes can have multiple names (e.g., a square is also a rectangle) in much the same way that individual numbers can have multiple names (e.g., $\frac{1}{2}$ is also $\frac{2}{4}$). They also realize that changing the layout of a given number of linear units can change the number of area units enclosed by the linear units and vice versa.

Attributes

- The student categorizes shapes based on shared attributes, focusing on quadrilaterals.

- The student solves real-world and mathematical problems involving perimeter.

CCSS-M

- 3.G.A.1, 3.MD.D.8

Additional Specifications

- Items involving perimeter may include identifying figures with the same perimeter and different areas or the same area and different perimeters. Thus, it is recommended that this strand be assessed after strand 9 or 10.
- Most items in this strand will involve quadrilaterals.
- Items assessing the analysis and categorization of shapes based on shared attributes are limited to rectangles, squares, rhombi, and generic "quadrilaterals."

Common Errors and Misconceptions

The following list gives examples of the common Grade 3 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student misinterprets meanings of and relationships among the four operations (e.g., subtracts instead of dividing).
- The student counts the 0-mark on a number line as the first mark (i.e., begins counting at the beginning of the first interval, rather than at the end).
- The student compares fractions based on the magnitude of the numeral in the denominator rather than the relative size of the unit partitions (e.g., decides $\frac{1}{2} < \frac{1}{3}$ because 2 < 3).
- The student compares fractions without regard to the necessity of a same-sized whole.
- The student double-counts the "overlap" area of a figure composed of two rectangles.
- The student does not recognize a whole number as being a collection of fractional parts (e.g., does not believe the measurement can be a whole number when measuring to the nearest half-inch).

Grade 4

General Discussion

Grade 4 testlets place an emphasis on the critical work of fourth grade as students:

- Generalize base-ten place value concepts for whole numbers.
- Build towards fluency with whole numbers in the four operations.
- Extend understanding of fraction equivalence.

- Compute with fractions.

In fourth grade, students build on their foundational understanding of hundreds, tens, and ones as they generalize the ten-to-one relationship between adjacent place values to include all whole number places. They use multiplicative reasoning to compare the value of a digit in one place to the value of the same digit one place to the right (e.g., recognizing that the value of a 2 in the thousands place is ten times as much as the value of a 2 in the hundreds place). Strand 1 focuses on concepts associated with base-ten whole number place value.

Fourth grade students apply their increasing understanding of place value along with models and the properties of operations as they continue to build toward fluency in the four operations. By the end of fourth grade, students are fluent in addition and subtraction of whole numbers. They expand their understanding of the distributive property of multiplication over addition as they multiply greater numbers, multiplying either two two-digit numbers or a number up to four digits by a one-digit number. Students also divide a number up to four digits by a one-digit number using strategies based on place value and the properties of operations. Strands 2, 4, and 5 focus on whole number operations.

Problems drawn from real-world contexts offer students the opportunity to think algebraically while performing calculations with whole numbers of all sizes. Students use a variety of modeling and reasoning skills to understand and solve a problem and to decide whether the answer makes sense in the context. Strand 6 focuses on problem-solving in real-world contexts, including problems that involve numeric patterns.

In fourth grade, students extend their previous understanding of fraction equivalence, recognizing that the numerators and denominators of equivalent fractions are related by the same factor. They apply previous understanding of fractions as numbers to recognize that the properties of operations for whole numbers also apply to fraction operations. They recognize a non-unit fraction as the sum of unit fractions (e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$), analogous to how a whole number greater than 1 is equal to a sum of ones (e.g., $3 = 1 + 1 + 1$). They apply this understanding to add and subtract fractions with like denominators, and they begin to combine their knowledge of equivalent fractions with fraction addition as they add a fraction with the denominator 10 to a fraction with the denominator 100. Students also apply their previous understanding of multiplication as repeated addition to multiply a fractional quantity by a

whole number (e.g., recognizing that $3 \times \frac{4}{5}$ means three groups of $\frac{4}{5}$ and is equivalent to $\frac{4}{5} + \frac{4}{5} + \frac{4}{5}$). Strands 7–10 focus on fractions.

Students taking grade 4 testlets do not have access to a calculator or reference sheet.

Strands

Strand 1: Place Value in the Base Ten System

Students extend previous work with whole number place value concepts to include all whole numbers. They describe the relative value of the same digit in adjacent places using multiplicative comparison, recognizing that a ten thousand is ten times as great as a thousand, a thousand is ten times as great as a hundred, and so on. They use place value understanding to read, write, compare, and round whole numbers in numeric, expanded, and word forms.

Attributes

- The student applies an understanding of the 10-to-1 multiplicative relationship between a digit in one place and the same digit in the place to its immediate left.
- The student reads, writes, compares, and rounds multi-digit whole numbers.

CCSS-M

- 4.NBT.A.1, 4.NBT.A.2, 4.NBT.A.3

Additional Specifications

- Items will use a balance of word form, number form, and expanded form of numbers.
- Items will use only whole numbers.
- Items will only reference the relationship from a greater place value to a lesser place value (i.e., that the place value on the left has a value that is 10 times as great as the place value on the right).

Strand 2: Addition and Subtraction Algorithms

Students use a standard algorithm to demonstrate fluency in multi-digit addition and subtraction.

Attributes

- The student fluently adds multi-digit whole numbers using a standard algorithm.
- The student fluently subtracts multi-digit whole numbers using a standard algorithm.

CCSS-M

- 4.NBT.B.4

Additional Specifications

- Items will use only whole numbers with sums and minuends less than 1,000,000.

- Most items will be at the higher end of the allowable range of numbers.
- Most items will include multiple regroupings across place values.
- Most items in the strand will be pure math (no real-world context). In items that do include a real-world context, the goal is to provide the student with a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.

Strand 3: Extend Concepts of Multiplication

Students extend their previous understanding of multiplication, recognizing it as a method for comparing two quantities. They multiply or divide to solve one-step problems involving multiplicative comparison. Students also apply multiplication understanding to classify numbers as prime or composite and to identify factor pairs and multiples of whole numbers.

Attributes

- The student interprets and solves problems involving multiplicative comparison.
- The student identifies factor pairs and multiples of whole numbers and determines whether a number is prime or composite.

CCSS-M

- 4.OA.A.1, 4.OA.A.2, 4.OA.B.4

Additional Specifications

- Items will use only whole numbers in the strand.
- Items assessing factor pairs, prime, and composite will only use whole numbers less than or equal to 100.
- Items assessing multiples will only use whole numbers less than or equal to 10 as the target number (i.e., only assess multiples of the numbers 1–10 inclusive).

Strand 4: Multi-Digit Multiplication

Students apply place value understanding and properties of operations to multiply greater numbers. They decompose one or both factors by place value and apply the distributive property, using area models and equations to illustrate and explain the work.

Attribute

- The student multiplies up to four-digit by one-digit whole numbers and two two-digit numbers.
- The student illustrates and explains multi-digit multiplication.

CCSS-M

- 4.NBT.B.5

Additional Specifications

- Items will be balanced between multiplying a two-, three-, or four-digit number by a one-digit number (and vice versa) and multiplying a two-digit number by a two-digit number.
- Models will include area models and equations.
- Most items in the strand will be pure math (no real-world context). In items that do include a real-world context, the goal is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.

Strand 5: Multi-Digit Division

Students apply place value understanding and properties of operations to divide greater numbers. They decompose the dividend by place value and apply the distributive property, using area models and equations to illustrate and explain the work.

Attributes

- The student finds whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
- The student illustrates and explains multi-digit division.

CCSS-M

- 4.NBT.B.6

Additional Specifications

- Items will be balanced between two-, three-, and four-digit dividends. All items will have a one-digit divisor.
- Models will include area models and equations.
- Most items in the strand will be pure math (no real-world context). In items that do include a real-world context, the goal is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving. Contexts will vary between partitive and quotative situations.
- Remainders will be represented using “R” notation (e.g., 3 R2).

Strand 6: Real-World Problems and Patterns

Students apply their conceptual understanding of and skill with the four operations to solve multistep problems that are drawn from real-world contexts. They assess the reasonableness of their answer and interpret remainders in the context of the problem. Students use operations and algebraic reasoning to identify features of a number or shape pattern that are not identified in the pattern rule.

Attributes

- The student solves multistep word problems using the four operations, including problems where a remainder must be interpreted and problems involving multiplicative comparison.
- The student generates number and shape patterns and identifies features of the pattern.

CCSS-M

- 4.OA.A.3, 4.OA.C.5

Additional Specifications

- Multistep items will include at least three distinct operational steps. Interpreting a remainder can replace one operational step where appropriate.
- Items will use only whole numbers.
- All calculations will be limited to grade-level operations on whole numbers.
- Items assessing understanding of reasonableness will focus on critiquing a response generated by a character in the problem.
- Items assessing interpretation of a remainder will focus on whether the remainder should be ignored/discarded (e.g., the answer is the quotient), reported as is (e.g., the answer is the remainder), or included in the response (e.g., the answer is the quotient rounded up to the next whole number).
- Items assessing patterns will provide the pattern rule in the item stem.
- Most items assessing generating patterns will focus on identifying a future term, rather than the term(s) that immediately follow any given portion of the sequence.

Strand 7: Compare and Find Equivalent Fractions

Students extend their understanding of equivalent fractions and more formally explain the equivalency, recognizing that the same number can have multiple names that vary based on the number and size of parts that comprise it. Students compare fractions with unlike numerators and unlike denominators using this understanding, as well as benchmarks, number line models, and equivalent fractions with like denominators.

Attributes

- The student recognizes and generates equivalent fractions and explains the equivalency using fraction models.
- The student compares fractions.

CCSS-M

- 4.NF.A.1, 4.NF.A.2

Additional Specifications

- Fractions are limited to denominators of 1, 2, 3, 4, 5, 6, 8, 10, 12, and 100. Incorrect answer options may include other denominators as indicated by the needs of the individual item.
- Items using benchmark numbers to compare fractions are limited to benchmarks that are whole numbers of halves.

Strand 8: Add and Subtract Fractions

Students extend their understanding of addition and subtraction to include applying these operations to fractions. They see a non-unit fraction as the sum of unit fractions and use this understanding to add and subtract fractions with the same denominator, in mathematical and real-world problems. Students retrieve fractional measurement data from line plots to provide additional context for fraction operations.

Attributes

- The student understands a fraction as the sum of unit fractions with the same denominator and uses this understanding to add and subtract fractions (including fractions greater than 1 and mixed numbers).
- The student solves real-world problems by adding and subtracting fractions with like denominators, including problems with fractional data presented on a line plot.

CCSS-M

- 4.NF.B.3, 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c, 4.NF.B.3.d, 4.MD.B.4

Additional Specifications

- Fractions are limited to denominators of 1, 2, 3, 4, 5, 6, 8, 10, 12, and 100. Incorrect answer options may include other denominators as indicated by the needs of the individual item.
- Real-world problems will be distributed across the problem types given in Appendix B.

Strand 9: Multiply Fractions

Students extend their understanding of multiplication and multiples to include repeated addition of fractional quantities. They multiply a fractional quantity by a whole number to solve mathematical and real-world problems, representing the multiplication with a visual model or equation.

Attributes

- The student applies knowledge of multiples to multiply a fraction by a whole number.
- The student solves word problems by multiplying a fraction by a whole number.

CCSS

- 4.NF.B.4, 4.NF.B.4.a, 4.NF.B.4.b, 4.NF.B.4.c

Additional Specifications

- Fractions are limited to denominators of 1, 2, 3, 4, 5, 6, 8, 10, 12, and 100. Incorrect answer options may include other denominators as indicated by the needs of the individual item.
- Interpretation of multiplication is limited to situations involving a whole number of groups, each of a fractional size. For example, an item may ask students to find the total weight of grain in 4 bags of grain with each bag weighing $\frac{3}{8}$ of a pound. However, an item will not ask students to find the weight of $\frac{3}{8}$ of a bag of grain given that a full bag weighs 4 pounds.
- Real-world problems will be distributed across the problem types given in Appendix C.

Strand 10: Decimal Fractions

Students relate decimal fractions that are written in fraction form and decimal form by translating between the forms. They use concepts of fraction comparison to compare decimal fractions written in decimal form. They add a fraction with denominator 10 and a fraction with denominator 100 by rewriting the former as a fraction with denominator 100, using visual models and equations to illustrate the addition.

Attributes

- The student uses fraction understanding to compare decimals.
- The student expresses fractions as decimals and adds decimal fractions.

CCSS-M

- 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

Additional Specifications

- Fractions are limited to denominators of 10 and 100. Incorrect answer options may include other denominators as indicated by the needs of the individual item.

- Items do not assess comparing decimals based on place value understanding. Comparisons are made based on understanding of comparing fractions and transferring that comparison to the decimal forms.

Strand 11: Solve Measurement Problems

Students use multiplicative comparison to describe the relative size of measurement units within the same system of measurement and multiply to find the number of smaller units that are equivalent to a given number of larger units. They convert between measurement units as part of their work to solve real-world and mathematical measurement problems. Students also solve problems by applying the formulas for the area and perimeter of a rectangle.

Attributes

- The student solves real-world and mathematical measurement problems, including problems involving relative sizes of measurement units.
- The student solves problems using the area and perimeter formulas for rectangles.

CCSS-M

- 4.MD.A.1, 4.MD.A.2, 4.MD.A.3

Additional Specifications

- Items may include fractions, limited to denominators of 1, 2, 3, 4, 5, 6, 8, 10, 12, and 100. Incorrect answer options may include other denominators as indicated by the needs of the individual item.
- Items assessing measurement conversions leverage the idea that the larger unit is a group comprised of a certain number of the smaller unit (e.g., converting from the larger unit to the smaller unit is accomplished by recognizing that 3 pounds is the same as 3 groups of 16 ounces each, or recognizing that 1 pound is 16 times as much as 1 ounce). Items will not use “conversion rates.”
- Items may include decimals, limited to tenths and hundredths. Students will not be asked to compute with the decimals but may be directed to express the decimals in fraction form as part of solving the problem.
- Items do not assess students’ ability to substitute values into the area or perimeter formula. Items will assess students’ ability to use the relationships between and among length, width, perimeter, and area to solve problems involving these measures.

Strand 12: Angles and Geometry

Students recognize angles as figures which can be measured using unit degrees. They recognize that angle measure is additive; when two angles are joined, the measure of the two angles combined is equal to the sum of the measure of each angle. Students also recognize

angles as geometric shapes, along with points, line segments, rays, and lines, and they classify shapes according to their geometric properties.

Attributes

- The student understands concepts of angle and measure angles.
- The student draws and identifies lines and angles and classifies shapes by properties of their lines and angles.

CCSS-M

- 4.G.A.1, 4.G.A.2, 4.G.A.3, 4.MD.C.5, 4.MD.C.5.a, 4.MD.C.5.b, 4.MD.C.6, 4.MD.C.7

Additional Specifications

- Items may assess student understanding that *n* degrees is equivalent to $\frac{n}{360}$ of a circle.
- In items where the fraction $\frac{n}{360}$ is to be simplified, the denominator of the equivalent fraction is limited to 1, 2, 3, 4, 5, 6, or 8.

Common Errors and Misconceptions

The following list gives examples of the common Grade 4 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student confuses additive and multiplicative comparison.
- The student compares numbers by looking at the first digit of the two numbers without consideration of place value (e.g., decides 62,058 > 114,520 because 6 > 1).
- The student disregards place value when determining partial products in a multiplication problem (e.g., includes a partial product of 8 × 3, instead of 80 × 30, when multiplying 87 × 35).
- The student interprets the remainder incorrectly.
- The student counts the 0-mark on a number line as the first mark (i.e., begins counting at the beginning of the first interval, rather than at the end).
- The student compares fractions without regard to the relative size of the wholes.
- The student adds the numerators and adds the denominators.
- The student multiplies both the numerator and the denominator by the whole number.
- The student uses an incorrect relationship between measurement units to convert.
- The student considers an angle measure to be dependent on the distance of the arc from the vertex.

Grade 5

General Discussion

Grade 5 testlets place an emphasis on the critical work of fifth grade as students:

- Generalize base-ten place value concepts for decimals.
- Begin decimal computation.
- Add and subtract fractions with unlike denominators, multiply two fractions, or divide a whole number by a unit fraction (and vice versa).
- Understand volume as a three-dimensional measurement attribute and recognize its relationship to multiplication and division.

In fifth grade, students finalize their understanding of place value concepts in the base-ten system. They extend previous work with the ten-to-one relationship between adjacent places into the decimal places and recognize the inverse one-to-ten relationship. They also begin to work with exponential form, specifically when considering powers of ten as they relate to place value. Strand 2 focuses on place value and the powers of ten.

Building on previous work with decimal fractions and place value, students in grade 5 begin to work with decimal computation. Students use models, properties of operations, and strategies based on place value to expand their understanding of operations with whole numbers to recognize that operations with decimals follow the same rules (e.g., recognizing that in adding two decimals, one adds tenths to tenths, hundredths to hundredths, and so on). Strand 5 focuses on decimal operations.

Fifth-grade students leverage their previous understanding of equivalent fractions as they add and subtract fractions with unlike denominators, rewriting each as an equivalent fraction with like denominators to complete the computation. They use number sense to assess the reasonableness of their sum or difference, taking into consideration the closeness of each fraction to a common benchmark number. Strand 6 focuses on addition and subtraction of fractions.

Students also extend previous work with multiplication, particularly fraction multiplication, as they multiply a whole number by a fraction or multiply two fractions, including mixed numbers. They apply an understanding of multiplication as a comparison to develop a deeper understanding of number, particularly recognizing that the value of one factor, relative to 1, determines whether the product will be greater than, equal to, or less than the other factor. Combining this concept with previous understanding of multiplicative comparison, students

recognize multiplication by a fraction as scaling, laying the foundation for future work with scale factors. Strand 7 focuses on multiplication of fractions.

Students extend their understanding of division as they divide unit fractions by whole numbers and vice versa. They use models to interpret the calculation. In a similar vein, students come to recognize that a fraction can be interpreted as the division of the numerator by the denominator. They divide whole numbers and express the quotient and/or remainder as a fraction, representing a new way to interpret remainders in real-world problems. Strand 8 focuses on these two concepts associated with fractions and division.

In fifth grade, students continue to grow their understanding of geometric measurement, moving into the third dimension as they explore concepts of volume. They recognize volume as the measure of the amount of space enclosed by a three-dimensional figure and understand that cubic units are composed of length units expressed in three dimensions. They explore different methods of finding the volume of a right rectangular prism, or a figure composed of the same, including packing the prism with unit cubes, multiplying the area of the base by the height, or multiplying the three edge lengths. Working with volume provides a context in which students are able to practice and build toward fluency with whole number multiplication, as well as interpret and evaluate numerical expressions that represent a three-fold product. Strands 9–10 focus on volumetric measurement.

Students in grade 5 also develop a foundational understanding of a variety of concepts that will support work in later grades. Interpreting and evaluating numerical expressions lays the groundwork for algebraic expressions and, eventually, equations. Strand 1 focuses on numerical expressions. Students also begin to understand the Cartesian coordinate system by arranging two number lines perpendicular to each other and naming the location of a point within the plane according to its distance from 0 on each line. In later grades, students will extend their work to include negative coordinate values. Strand 11 focuses on the first quadrant of the coordinate plane. Fifth grade mathematics also provides students with the time and space to continue progressing through the levels of geometric understanding as defined by van Hiele (Fuys, Geddes, and Tischler, 1988), coming to recognize the hierarchical relationships among classes of geometric figures. Strand 12 focuses on the attributes of two-dimensional figures.

Students taking grade 5 testlets do not have access to a calculator and do have access to a reference sheet.

Strands

Strand 1: Numerical Expressions

Students apply the standard order of operations to evaluate numerical expressions. Students compare the values of two expressions, especially those that differ only by the presence or absence of grouping symbols. They translate an expression between word form and numeric/symbolic form.

Attributes

- The student evaluates numeric expressions.
- The student reads, writes, and interprets numeric expressions.

CCSS-M

- 5.OA.A.1, 5.OA.A.2

Additional Specifications

- Expressions do not exceed one level of grouping, which can be represented using parentheses, braces, or brackets.
- Items may include whole numbers and fractions.
- Calculations in items are limited to expectations established in grade 4 standards.
- Some items will include a real-world context. The goal of this is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.

Strand 2: Place Value and Powers of Ten

Students explain the multiplicative relationship between adjacent place values in whole and decimal numbers. They demonstrate that a digit has a value that is 10 times as much as the value of the same digit in the place to its right and $\frac{1}{10}$ the value of the same digit in the place to its left. Students identify and explain patterns related to place value and the location of digits relative to the decimal point when multiplying or dividing a number by a power of ten, which they represent in exponent form.

Attributes

- The student applies understanding of the 10-to-1 multiplicative relationship between adjacent places in a number.
- The student explains patterns related to place value when multiplying or dividing by a power of ten and evaluates powers of ten represented in exponent form.

CCSS-M

- 5.NBT.A.1, 5.NBT.A.2

Additional Specifications

- Items assessing the relationship between places are limited to using the same digit in immediately adjacent place value places. The digit may be repeated within a single number or may be given in two separate numbers.
- In alignment with the major work of grade 5, most items will focus on the aspect of this strand that represents an extension from previous grades: understanding the right-to-left relationship (a digit has $\frac{1}{10}$ of the value as the same digit located one place to the left) and working in decimal places.
- Exponents are limited to whole numbers less than or equal to 6.
- Items involving calculation with exponential powers of ten are limited to multiplying or dividing a single number by a single power of ten.

Strand 3: Represent and Compare Decimals

Students translate numbers that extend to the thousandth place among numeric, word, and expanded forms. They use place value understanding to round decimals. Students compare and order decimals based on concepts of place value.

Attributes

- The student reads, writes, and rounds decimals to the thousandths place.
- The student uses place value understanding to compare and order decimals.

CCSS-M

- 5.NBT.A.3, 5.NBT.A.3.a, 5.NBT.A.3.b, 5.NBT.A.4

Additional Specifications

- Some items may include measurement contexts limited to metric measurement units. The goal of this is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving.
- Items that assess rounding are limited to rounding a decimal to the nearest whole number, tenth, and hundredth.
- Items that assess ordering will be limited to no more than four numbers to put in order.

Strand 4: Multiply and Divide Whole Numbers

Students multiply whole numbers using a standard algorithm and divide whole numbers using strategies based on place value and the properties of operations. They apply these skills while solving one-step real-world problems.

Attributes

- The student multiplies whole numbers using the standard algorithm and applies multiplication skills to solve real-world problems.

- The student divides whole numbers using a variety of strategies based on place value and the properties of operations and applies division skills to solve real-world problems.

CCSS-M

- 5.NBT.B.5, 5.NBT.B.6

Additional Specifications

- Items assessing multiplication are limited to a total of six digits in the factors and will focus on four-digit by two-digit and three-digit by three-digit calculations.
- Items assessing division are limited to four-digit dividends and two-digit divisors.
- Items assessing division may include quotients with or without remainders. Students are not expected to interpret remainders in these items.
- Items including real-world problems will emphasize the calculation rather than the problem-solving and are limited to one step.
- Some items will include a real-world context. The goal of this is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving. In these items, standard 5.MD.A.1 will inform the contexts selected. Measurement conversions will be limited to those assessed in Grade 4 or those which are provided on the Grade 5 Reference Sheet.

Strand 5: Operations with Decimals

Students extend their work with the four operations in the base-ten system to include operations with decimals. They use strategies (including visual models) based on place value or properties of operations to add, subtract, multiply, and divide decimals.

Attributes

- The student adds and subtracts decimals to the hundredths.
- The student multiplies and divides decimals to the hundredths.

CCSS-M

- 5.NBT.B.7

Additional Specifications

- Items assessing addition will be limited to addends to the hundredths.
- Items assessing subtraction will be limited to numbers that correspond with the limitations on addition.
- Items assessing multiplication will be limited to factors to the tenths such that the product will be limited to the hundredths. Factors will include no more than six digits in

the factors in total (a zero in the ones place of a decimal less than 1 is excluded from the count of digits).

- Items assessing division will be limited to numbers that correspond with the limitations on multiplication. Dividends are limited to four digits while divisors are limited to two digits.
- Some items will include a real-world context. The goal of this is to provide the student a relatable entry point to demonstrate understanding of the mathematics, not to assess problem-solving. In these items, standard 5.MD.A.1 will inform the contexts selected. Measurement conversions will be limited to those assessed in Grade 4 or those which are provided on the Grade 5 Reference Sheet.

Strand 6: Add and Subtract Fractions

Students extend their understanding of fraction computation to add and subtract fractions with unlike denominators, recognizing the need for fractions to refer to the same whole. They rewrite one or more fractions as equivalent fractions with like denominators to add or subtract. They use benchmark fractions, number sense, and models to estimate the solution to a real-world problem and determine the reasonableness of the solution based on the estimate.

Attributes

- The student adds and subtracts fractions with unlike denominators.
- The student applies fraction addition and subtraction skills to solve real-world problems and assess the reasonableness of the solution.

CCSS-M

- 5.NF.A.1, 5.NF.A.2, 5.MD.B.2

Additional Specifications

- Items assessing problem-solving will include fractions limited to denominators that make sense in the context of the problem (e.g., using halves for customary unit measurements or tenths for metric unit measurements).
- Items will not include unwieldy denominators.
- Items may ask students to evaluate the use of various strategies, such as benchmark numbers, to determine the reasonableness of a solution.
- Some items will include a real-world measurement or data context, where students will need to retrieve fractional data presented in a line plot display to solve the addition or subtraction problem.
- Mathematical items do not require students to simplify answers.

Strand 7: Multiply Fractions

Students extend their understanding of multiplication to include multiplying fractions, including mixed numbers. They multiply fractional side lengths to find the area of a rectangle and find the quantity that represents a fraction of a fractional quantity. Students describe the relationship between the values of a factor and product based on the value of the other factor and use this relationship to explain equivalent fractions. They use these skills to estimate and solve real-world problems and determine the reasonableness of their solution using their estimate.

Attributes

- The student interprets fraction multiplication in a variety of ways and applies this understanding to multiply fractions, mixed numbers, and whole numbers.
- The student applies fraction multiplication skills to solve real-world problems and assess the reasonableness of the solution.

CCSS-M

- 5.NF.B.4, 5.NF.B.4.a, 5.NF.B.4.b, 5.NF.B.5, 5.NF.B.5.a, 5.NF.B.5.b, 5.NF.B.6, 5.MD.B.2

Additional Specifications

- This strand will include items that represent multiplication in the mathematical contexts of equal groups, area of a rectangle, and scaling/comparison (including equivalent fractions).
- Items assessing problem-solving will include fractions limited to denominators that make sense in the context of the problem (e.g., using halves for customary unit measurements or tenths for metric unit measurements).
- Items will not include unwieldy denominators.
- Some items will include a real-world measurement or data context, where students will need to retrieve fractional data presented in a line plot display to solve the multiplication problem.
- Mathematical items do not require students to simplify answers.

Strand 8: Division with Fractions

Students extend their understanding of division to decompose a whole number into unit fractions with like denominators or to decompose a unit fraction into a whole number of lesser unit fractions. They solve real-world problems that involve interpreting the quotient of these computations. Students divide whole numbers and represent the result in fraction or mixed number form, understanding that a fraction represents the division of the numerator by the denominator.

Attributes

- The student applies understanding of division to divide a unit fraction by a whole number and a whole number by a unit fraction.
- The student solves real-world problems and assesses the reasonableness of the solution by dividing fractions and by representing the quotient of two whole numbers as a fraction.

CCSS-M

- 5.NF.B.3, 5.NF.B.7, 5.NF.B.7.a, 5.NF.B.7.b, 5.NF.B.7.c, 5.MD.B.2

Additional Specifications

- Items assessing division with fractions (i.e., items aligned to 5.NF.C.7) are limited to the division of a whole number by a unit fraction or a unit fraction by a whole number.
- Most items assessing division with fractions (i.e., items aligned to 5.NF.C.7) will focus on reasoning and interpretation rather than problem-solving.
- Items assessing problem-solving will include fractions limited to denominators that make sense in the context of the problem (e.g., using halves for customary unit measurements or tenths for metric unit measurements).
- Items will not include unwieldy denominators.
- Whole number division is limited to the expectations of Grade 4, where dividends may not exceed four digits and divisors must be a single digit.
- Some items will include a real-world measurement or data context, where students will need to retrieve fractional data presented in a line plot display to solve the division problem.
- Mathematical items do not require students to simplify answers.

Strand 9: Unit Cubes and Cubic Units

Students extend their understanding of measurement to include three-dimensional units, recognizing a unit cube as the measurement unit used to find the number of units that fill a figure. They understand that unit cubes must completely fill the shape without overlaps or gaps between cubes for the number to represent the volume, analogous to how unit squares or linear units must be aligned. Students count the number of unit cubes to find the volume of a right rectangular prism and relate this to multiplication, first multiplying the number of unit cubes in each layer by the number of layers and then multiplying the edge lengths or multiplying the area of the base by the height.

Attributes

- The student understands concepts of volume.

- The student demonstrates volume of a right rectangular prism can be found by counting unit cubes and by multiplying edge lengths.

CCSS-M

- 5.MD.C.3, 5.MD.C.3.a, 5.MD.C.3.b, 5.MD.C.4, 5.MD.C.5, 5.MD.C.5.a

Additional Specifications

- Items in this strand may include improvised units for volumetric measurement (e.g., cube-shaped erasers).
- Items will allow for the length, width, and height of a prism to be interchangeable.
- Items in this strand will reference the “edge lengths,” not “side lengths.”
- Most items will involve multiplication of the number of cubic units in a layer by the number of layers or of the edge lengths, rather than counting unit cubes.
- Edge lengths are restricted to whole numbers.
- Items in this strand are restricted to the concept of “packing” a prism with unit cubes, not “filling” the prism.

Strand 10: Solve Volume Problems

Students multiply edge lengths to find the volume of right rectangular prisms in mathematical and real-world contexts. They recognize that volume is additive; when two prisms are joined, the volume of the whole figure is equal to the sum of the volumes of the two prisms. Students use this understanding to find the volume of composite rectilinear prisms.

Attributes

- The student finds the volume of right rectangular prisms in mathematical and real-world problems.
- The student finds the volume of composite rectilinear figures in mathematical and real-world problems.

CCSS-M

- 5.MD.C.5, 5.MD.C.5.b, 5.MD.C.5.c

Additional Specifications

- This strand will include items that assess finding the volume of a right rectangular prism and the volume of a figure composed of two or three non-overlapping right rectangular prisms.
- Items will allow for the length, width, and height of a prism to be interchangeable.
- Items in this strand will reference the “edge lengths,” not “side lengths.”
- Edge lengths are restricted to whole numbers.

- This strand will include items that assess multiplying the area of the base of the figure by its height.
- This strand will include items that assess using the formula for the volume of a right rectangular prism.
- This strand may include items that ask students to provide one or more unknown dimensions for a given volume.
- Multiplication and division are restricted to general grade-level limitations.

Strand 11: Understand the First Quadrant

Students locate coordinate points in the first quadrant and identify the ordered pair that names a given coordinate point. They interpret the meaning of the *x*- and *y*-coordinates, especially in terms of how the coordinate indicates the distance of the point along one axis and away from the other. Students describe the relative location of points and find a point that is located a specific direction and distance from a given point. They create ordered pairs from corresponding terms in simultaneous patterns, locate the resulting points in the first quadrant, and describe the apparent relationship between the terms.

Attributes

- The student applies understanding of the structure of the coordinate plane.
- The student plots points on a coordinate plane and interprets the coordinate values in context.

CCSS-M

- 5.G.A.1, 5.G.A.2, 5.OA.B.3

Additional Specifications

- Items in this strand are limited to the first quadrant.
- Items in this strand are limited to whole number coordinates.
- Items aligned to 5.OA.B.3 will give the pattern rule in the stem.
- In alignment with the primary work of the grade, items aligned to 5.OA.B.3 will focus on identifying relationships between corresponding terms and plotting them on the coordinate plane, rather than simply identifying the terms.
- Students will be assessed on both interpretations of the meaning of the values in an ordered pair (i.e., the distance along one axis and the distance from the other axis).

Strand 12: Attributes of Geometric Figures

Students use geometric attributes of two-dimensional figures to determine all the classes that a shape belongs to. They identify whether the members of given classes always, sometimes, or

never have various attributes. Students identify which classes of shapes are subclasses of other classes.

Attributes

- The student arranges two-dimensional figures into a hierarchy based on geometric properties.
- The student demonstrates that geometric attributes of one class of shapes belong to all subclasses of that shape.

CCSS-M

- 5.G.B.3, 5.G.B.4

Additional Specifications

- Items in this strand focus on the hierarchy of classes of quadrilaterals.
- Items in this strand primarily align to level 2 on the van Hiele theory of geometric thought.
- Items in this strand do not use hash marks to indicate congruency.

Common Errors and Misconceptions

The following list gives examples of the common Grade 5 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student reverses the ten-to-one relationship between adjacent place values.
- The student always evaluates expressions from left to right, ignoring the order of operations.
- The student counts the number of zeros in the product and uses that quantity as the exponent on the power of ten.

Grade 6

General Discussion

Grade 6 testlets emphasize the critical work of sixth grade as students:

- Connect multiplication and division to ratio and rate and use ratio reasoning to solve problems.
- Build an understanding of negative numbers and deepen the understanding of operations with rational numbers to divide fractions by fractions.
- Write, interpret, and use algebraic expressions, equations, and inequalities.

- Develop an understanding of statistical variability and display and describe data distributions.
- Reason about relationships among shapes to determine area, surface area, and volume.

In sixth grade, students begin to develop an understanding of ratios, unit rates, and proportional relationships. Building on previous work with multiplication and division, they use ratio reasoning to solve problems. Strands 1 and 2 focus ratios and rates.

Students in grade 6 extend their understanding of the number system to include numbers less than 0. They understand negative numbers in context, compare and order rational numbers, and reason about the location of negative rational numbers on a number line and coordinate grid. Strands 5 and 11 focus on rational numbers and the coordinate plane.

Students in sixth grade also extend their previous understanding of division to divide with fractions. Strand 3 focuses on dividing fractions. They achieve fluency in the four operations on positive rational numbers, using standard algorithms to accurately and efficiently complete straightforward computations. Strand 4 focuses on computational fluency.

In sixth grade, students use their knowledge of arithmetic and the properties of operations to write and interpret algebraic expressions, equations, and inequalities, evaluate algebraic expressions, and solve equations and inequalities. Students recognize equivalent expressions and use properties of operations to rewrite expressions and solve one-step equations. They understand that the solution to an equation or inequality is the value or set of values that makes an equation or inequality true. They write expressions and equations to represent a given situation. Students also understand the relationship between independent and dependent variables and use equations with two variables to represent relationships between quantities. Strands 6, 7, 8, and 9 focus on algebraic expressions, equations, and inequalities.

Students in sixth grade build upon their number sense to begin thinking statistically. Students understand that measures of center and variability are used to describe and summarize data sets. They display data sets in dot plots, histograms, and box plots, as well as compare and summarize data sets. Strand 12 focuses on statistical thinking.

In sixth grade, students build upon their knowledge of shapes to find area, surface area, and volume. They extend their work with rectangles to find the area of various polygons and develop formulas for the area of triangles and parallelograms. They use 2-dimensional nets to

find the surface area of prisms and pyramids and extend their understanding of volume to right prisms with fractional side lengths.

Students taking grade 6 testlets have access to a calculator for selected testlets. They also have access to the Grade 6 Student Reference Sheet.

Strands

Strand 1: Concepts of Ratios and Unit Rates

Students create ratios from information given in a table, a verbal description, as points on a graph, and in diagrams. They use these ratios to calculate a unit rate or solve a problem. Students compare ratios and unit rates to answer questions posed in word problems.

Attributes

- The student finds and compares unit rates and ratios.
- The student uses ratios and unit rates to solve problems.

CCSS-M

- 6.RP.A.1, 6.RP.A.2, 6.RP.A.3, 6.RP.A.3.a, 6.RP.A.3.b

Additional Specifications

- n/a

Strand 2: Percents and Measurement Conversions

Students calculate using percentages, finding the unknown total, unknown part, or unknown percentage, when given the other two values. They relate percentages to ratios. They solve real-world and mathematical problems that require conversion between measurement units, within and across systems.

Attributes

- The student solves problems involving percentages.
- The student uses ratio reasoning to convert between two systems of measurement.

CCSS

- 6.RP.A.3, 6.RP.A.3.c, 6.RP.A.3.d

Additional Specifications

- n/a

Strand 3: Divide Fractions

Students interpret division with fractions and represent real-world contexts with fraction division expressions. They use visual fraction models to explain the interpretation and representation. They divide fractions to solve mathematical and real-world problems.

Attributes

- The student interprets and computes quotients of fractions.
- The student solves word problems by dividing fractions and interprets the quotients where appropriate.

CCSS-M

- 6.NS.A.1

Additional Specifications

- Students may not use a calculator for this strand.

Strand 4: Computational Fluency

Students divide multi-digit whole numbers and represent non-integer results using fraction or decimal notation. They add, subtract, multiply, and divide decimals.

Attributes

- The student fluently divides multi-digit numbers using the standard algorithm.
- The student fluently adds, subtracts, multiplies, and divides multi-digit decimals using the standard algorithm for each operation.

CCSS-M

- 6.NS.B.2, 6.NS.B.3

Additional Specifications

- Students may not use a calculator for this strand.
- In alignment with the expectation of fluency, items in this strand should include numbers that require an efficient or standard algorithm to compute.
- Items assessing division of whole numbers are limited to five-digit dividends and two-digit divisors.
- Items assessing addition of decimals are limited to five-digit addends to the thousandths place, less than 100. Items assessing subtraction are limited to numbers that correspond to the limitations on addition.
- Items assessing multiplication of decimals are limited to factors to the thousandths place and have a maximum of seven digits combined. Items assessing division are limited to numbers that correspond to the limitations on multiplication.

Strand 5: Rational Numbers and Absolute Value

Students locate and identify rational numbers on a number line. They interpret rational numbers in context. They evaluate the absolute value of rational numbers and compare rational numbers, including comparing absolute values.

Attributes

- The student represents positive and negative values in context and represents rational numbers as points on a number line.
- The student understands and interprets absolute value and compares and orders rational numbers.

CCSS-M

- 6.NS.C.5, 6.NS.C.6, 6.NS.C.6.a, 6.NS.C.6.c, 6.NS.C.7, 6.NS.C.7.a, 6.NS.C.7.b, 6.NS.C.7.c, 6.NS.C.7.d

Additional Specifications

- n/a

Strand 6: Algebraic Expressions and Exponents

Students write and interpret expressions using variables to represent mathematical and real-world statements. They use substitution to evaluate these expressions. Students represent and evaluate expressions with exponents.

Attributes

- The student reads and writes expressions using numbers and variables.
- The student evaluates expressions including exponents and variables.

CCSS-M

- 6.EE.A.1, 6.EE.A.2, 6.EE.A.2.a, 6.EE.A.2.b, 6.EE.A.2.c

Additional Specifications

- n/a

Strand 7: Equivalent Expressions

Students apply properties of operations, find greatest common factors, and use the order of operations to identify and generate equivalent expressions using variables.

Attributes

- The student uses properties of operations to simplify expressions.
- The student generates and identifies equivalent expressions.

CCSS-M

- 6.EE.A.3, 6.EE.A.4, 6.NS.B.4

Additional Specifications

- n/a

Strand 8: Variables in Expressions and Equations

Students write expressions and equations using variables to represent mathematical and real-world statements. They use substitution to evaluate expressions and solve equations. They identify dependent and independent variables and write equations to represent relationships between the variables.

Attributes

- The student writes and uses expressions and equations to solve real-world and mathematical problems, using variables to represent unknown quantities.
- The student understands the relationship between independent and dependent variables and represents it in an equation.

CCSS-M

- 6.EE.B.5, 6.EE.B.6, 6.EE.B.7, 6.EE.C.9

Additional Specifications

- n/a

Strand 9: Write and Interpret Inequalities

Students identify solutions to strict and non-strict inequalities and graph the solutions on number lines. They write inequalities to represent real-world contexts. Students use substitution and knowledge of rational numbers to solve inequalities, including compound inequalities, involving positive and negative numbers.

Attributes

- The student demonstrates the understanding that the solution to an inequality is a value that makes the statement true.
- The student writes inequalities to represent constraints in real-world or mathematical problems and graphs the solutions on number line diagrams.

CCSS-M

- 6.EE.B.5, 6.EE.B.8

Additional Specifications

- Numbers used in this strand are limited to positive integers.

Strand 10: Solve Problems with Area and Volume

Students find the area of triangles and polygons composed of triangles and rectangles. They represent right prisms and pyramids with two-dimensional nets to find the surface area of the

figure. They multiply non-integer edge lengths to find the volume of right rectangular prisms. Students solve real-world problems involving surface area and volume of prisms.

Attributes

- The student finds the area of polygons and the surface area of three-dimensional shapes.
- The student finds the volume of right rectangular prisms.

CCSS-M

- 6.G.A.1, 6.G.A.2, 6.G.A.4

Additional Specifications

- n/a

Strand 11: The Coordinate Plane

Students locate points in all four quadrants of the coordinate plane and name the four quadrants. They find the distance between points sharing a coordinate and use these lengths to find the areas of polygons on the plane. They understand that a point reflected across one axis will have one coordinate the same and the other coordinate its opposite.

Attributes

- The student solves real-world and mathematical problems by graphing in four quadrants on the coordinate plane.
- The student draws polygons in the coordinate plane and finds the length of horizontal and vertical line segments using coordinates.

CCSS-M

- 6.G.A.3, 6.NS.C.6, 6.NS.C.6.b, 6.NS.C.8

Additional Specifications

- Items assessing the distance between points may or may not include an image of the points on a coordinate grid.

Strand 12: Concepts of Statistics

Students recognize statistical questions. They represent numerical data on dot plots, box plots, and histograms and describe the data sets quantitatively and qualitatively. They describe how a change in the data set changes the centers of measure.

Attributes

- The student recognizes statistical questions and displays numerical data on a number line.
- The student summarizes data sets in relations to their context.

CCSS-M

- 6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5, 6.SP.B.5.a, 6.SP.B.5.b, 6.SP.B.5.c, 6.SP.B.5.d

Additional Specifications

- Measures of center and range in this strand are limited to mean, median, mode, range, lower quartile, upper quartile, and interquartile range.
- Data sets in this strand will be described in terms of symmetry, skew, gaps, outliers, and peaks.

Common Errors and Misconceptions

The following list gives examples of the common Grade 6 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student incorrectly applies an additive relationship when identifying equivalent ratios (e.g., identifying 5:7 as equivalent to 4:6).
- The student misunderstands value of negative numbers, identifying numbers with greater absolute values as having greater value (e.g., $-5 > 3$).
- The student incorrectly applies distribute property, only multiplying the first term in parenthesis (e.g., $3(x + 2) = 3x + 2$).
- The student does not include all faces of a three-dimensional figure when finding surface area.
- The student confuses mean and median, using them interchangeably.

Grade 7

General Discussion

Grade 7 testlets emphasize the critical work of seventh grade as students:

- Develop an understanding of proportionality to recognize and graph proportional relationships and solve multistep problems.
- Complete understanding of operations with rational numbers by including negative integers, recognizing rational numbers, and expressing them in different forms.
- Use properties of operations to write and interpret expressions and equations, as well as solve multistep equations and inequalities.

- Extend understanding of shapes to solve problems involving area, surface area, and volume, as well as supplementary, complementary, and vertical angles, and construct geometric shapes with given conditions.
- Use random samples to make inferences about and compare populations.

In seventh grade, students extend their understanding of ratios and unit rates to develop an understanding of proportionality. They solve various multi-step problems involving proportional relationships including percentages and scale drawings. Students recognize and represent proportional relationships in a graph, table, and equation, as well as find the constant of proportionality. Strands 1 and 2 focus on proportional understanding.

Seventh graders complete their understanding of operations with rational numbers to negative numbers. They use their understanding of negative values in context to develop and apply rules for adding, subtracting, multiplying, and dividing with negative numbers. Students recognize numbers as irrational and rational and express rational numbers as fractions, decimals, and percents. Strands 3, 4, and 8 focus on operations with rational numbers.

In grade 7, students use their knowledge of arithmetic to write and interpret algebraic expressions, as well as use properties of operations to recognize and write equivalent expressions. They write algebraic equations to solve problems and use properties of operations to solve multi-step equations. Students also solve problems that can be represented by a multi-step algebraic inequality. Strands 5, 6, and 7 focus on algebraic expressions, equations, and inequalities.

Seventh graders continue the study of geometric shapes, solving problems involving area, surface area, and volume of two- and three-dimensional objects composed of polygons and right prisms. Students find the area and circumference of circles and use them to solve problems. Students analyze angle and side length measurements of triangles to determine how many unique triangles can be constructed from given conditions. They solve problems involving supplementary, complementary, and vertical angles. Strands 9 and 10 focus on geometry.

In seventh grade, students extend their statistical thinking to make inferences about and compare populations, as well as recognize and use random sampling methods to collect data. They use probability to describe the likelihood of simple and compound events and represent sample spaces using various methods. Strands 11 and 12 focus on statistics and probability.

Students taking grade 7 testlets have access to a calculator for selected testlets. They also have access to the Grade 7 Student Reference Sheet.

Strands

Strand 1: Ratios and Proportional Relationships

Students use ratios to compute unit rates. They create proportions from information given in different representations. Students recognize a proportional relationship in a graph and interpret the meaning of points on the graph of a proportional relationship. They compare unit rates when presented in different representations.

Attributes

- The student computes and identifies unit rates from tables, graphs, and verbal descriptions.
- The student determines if a relationship is proportional and interprets points in context.

CCSS-M

- 7.RP.A.1, 7.RP.A.2, 7.RP.A.2.a, 7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.2.d

Additional Specifications

- n/a

Strand 2: Solve Problems with Ratio and Proportion

Students calculate the unknown value in a proportion. They set up proportions from information about scale drawings and then calculate the actual or scale value. Students use percentages and other ratios to solve problems. Students represent and identify proportional relationships with equations.

Attributes

- The student represents proportional relationships with equations and solves multistep problems.
- The student uses proportional relationships to solve ratio, percent, and scale drawing problems.

CCSS-M

- 7.G.A.1, 7.RP.A.3

Additional Specifications

- n/a

Strand 3: Add and Subtract Rational Numbers

Students add and subtract rational numbers and solve problems involving addition and subtraction of rational numbers in real-world contexts. They represent the addition and subtraction of rational numbers on number line diagrams in mathematical and real-world contexts. They rewrite addition and subtraction expressions in equivalent forms.

Attributes

- The student adds rational numbers and interprets sums.
- The student subtracts rational numbers and interprets differences.

CCSS-M

- 7.NS.A.1, 7.NS.A.1.a, 7.NS.A.1.b, 7.NS.A.1.c, 7.NS.A.1.d

Additional Specifications

- Students may not use a calculator for this strand.

Strand 4: Multiply and Divide Rational Numbers

Students multiply and divide rational numbers and solve problems involving multiplication and division of rational numbers in real-world contexts. They interpret products and quotients of rational numbers in real-world contexts. They write fractions as non-terminating decimals. They identify whether the product of rational numbers is positive or negative.

Attributes

- The student multiplies rational numbers and interprets products.
- The student divides rational numbers and interprets quotients.

CCSS-M

- 7.NS.A.2, 7.NS.A.2.a, 7.NS.A.2.b, 7.NS.A.2.c, 7.NS.A.2.d

Additional Specifications

- Students may not use a calculator for this strand.

Strand 5: Expressions with Rational Numbers

Students represent real-world contexts with unknown values using algebraic expressions. They identify equivalent algebraic expressions. They identify the greatest common factor of terms in an algebraic expression and factor an expression to its simplest form.

Attributes

- The student generates equivalent expressions using properties of operations.
- The student rewrites expressions to highlight particular quantities.

CCSS-M

- 7.EE.A.1, 7.EE.A.2

Additional Specifications

- n/a

Strand 6: Solving Equations

Students solve algebraic equations in mathematical and real-world contexts and identify the steps taken to solve an equation. They solve problems in real-world contexts that can be represented by equations and represent real-world contexts with equations.

Attributes

- The student solves word problems that can be represented by equations of the form $px + q = r$ and $p(x + q) = r$.
- The student solves equations of the form $px + q = r$ and $p(x + q) = r$ fluently.

CCSS-M

- 7.EE.B.4, 7.EE.B.4.a

Additional Specifications

- Items in this strand are limited to the use of rational numbers.

Strand 7: Solving Inequalities

Students represent real-world contexts using inequalities. They represent the solution set of inequalities on number line diagrams. They interpret the solution set of an inequality in a real-world context. They write inequalities to represent a verbal statement.

Attributes

- The student solves word problems that can be represented by inequalities of the form $px + q > r$ or $px + q < r$.
- The student interprets the solution set of an inequality in the context of the problem.

CCSS-M

- 7.EE.B.4, 7.EE.B.4.b

Additional Specifications

- n/a

Strand 8: Solve Problems with Rational Numbers

Students solve problems in mathematical and real-world contexts. They interpret the solutions to problems. They represent problems using equations. Students solve problems with up to three steps.

Attributes

- The student solves real-world problems with rational numbers.
- The student solves mathematical problems with rational numbers.

CCSS-M

- 7.EE.B.3, 7.NS.B.3

Additional Specifications

- n/a

Strand 9: Angle Relationships and Triangles

Students identify conditions that can result in a triangle. They recognize whether one triangle, more than one triangle, or no triangles can be drawn to fit a set of conditions. They write and solve equations to represent the measure of angles. Students use facts about related angles to find their measure.

Attributes

- The student constructs triangles and other geometric shapes with given conditions.
- The student finds an unknown angle measure in a figure by writing and solving simple equations and using facts about supplementary, complementary, vertical, and adjacent angles.

CCSS-M

- 7.G.A.2, 7.G.B.5

Additional Specifications

- n/a

Strand 10: Solve Problems with Geometric Figures

Students solve problems involving geometric measurements in mathematical and real-world contexts. They find the area of two-dimensional figures composed of quadrilaterals and triangles and the surface area of three-dimensional figures with polygonal faces. They find the volume of right rectangular prisms with rational number edge lengths. They identify two-dimensional figures that result from slicing a three-dimensional figure. Students find the circumference, area, radius, and diameter of circles and report these values as rational approximations or in terms of pi.

Attributes

- The student applies formulas for geometric measurement.
- The student solves real-world and mathematical problems involving two- and three-dimensional figures.

CCSS-M

- 7.G.A.3, 7.G.B.4, 7.G.B.6

Additional Specifications

- n/a

Strand 11: Measures of Center and Variability

Students identify sample sets to best represent a population. They make inferences about a population based on random sampling. They calculate and use measures of center and variability to informally compare two populations. Students retrieve data from dot plots and box plots.

Attributes

- The student makes inferences about a population based on sample data.
- The student uses measures of center and variability to draw informal comparisons between two populations.

CCSS-M

- 7.SP.A.1, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4

Additional Specifications

- n/a

Strand 12: Probability

Students calculate the probability of a chance event. They identify the number of outcomes in a sample space for a compound event. They qualitatively interpret a quantitative reporting of probability. Students distinguish between theoretical and experimental probabilities.

Attributes

- The student approximates the probability of a chance event using a probability model or observed frequencies.
- The student represents and uses sample spaces for compound events to find the probability of a compound chance event.

CCSS-M

- 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.7.a, 7.SP.C.7.b, 7.SP.C.8, 7.SP.C.8.a, 7.SP.C.8.b

Additional Specifications

- n/a

Common Errors and Misconceptions

The following list gives examples of the common Grade 7 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student reverses the meaning of quantities or variables when representing a real-world context.
- The student uses additive relationships to describe proportions instead of multiplicative relationships.
- The student gives absolute value of subtraction problems involving negative integers (e.g., $8 - 12 = 4$).
- The student does not use inverse operations when solving equations.
- The student confuses supplementary angles with complementary angles, using them interchangeably.
- The student misunderstands the meaning of representative samples.

Grade 8

General Discussion

Grade 8 testlets emphasize the critical work of eighth grade as students:

- Apply properties of integer exponents.
- Develop an understanding of functions.
- Use algebraic thinking to solve multistep linear equations in one variable.
- Make connections between proportional relationships and linear equations.
- Understand the solution to a system of equations and use a variety of methods to solve systems of equations.
- Understand similarity, congruency, and geometric transformations on two-dimensional figures.
- Develop an understanding of the Pythagorean theorem.

In eighth grade, students extend their understanding of the number system to recognize irrational numbers. Students express rational numbers as decimals, fractions, and percents. They approximate the value of irrational numbers to compare them and plot them on a number line. Students extend their knowledge of operations to exponents and roots, applying exponent rules to produce equivalent equations and finding the square and cube roots of perfect squares and cubes. They understand that scientific notation can be used to express large and small numbers and they use scientific notation to solve problems.

Eighth graders build upon their knowledge of independent and dependent variables to understand functional relationships. They recognize functions represented as graphs, tables,

descriptions, and ordered pairs. Students connect the equation of a straight line to a linear function and describe the relationship between two quantities represented in a graph, as well as sketch graphs of functions based on verbal descriptions.

In grade eight, students continue to use their knowledge of arithmetic and algebra to solve multistep equations with one variable and rational coefficients. They understand and recognize equations with different types of solutions.

In eighth grade, students build upon their previous understanding of proportionality to graph proportional relationships and compare relationships represented in different ways. They connect the constant of proportionality with the slope of a line and use it to describe and compare relationships. Students use similar triangles graphed on a coordinate grid to find the slope of line segments on the same line and explain why they are always the same.

Eighth graders extend their understanding of linear equations with two variables to systems of equations. They recognize the intersection of two lines as the solution to the system of equations, recognize systems of equations with different types of solutions, and use the graphs of lines to estimate solutions. Students solve simple systems of equations given algebraically and verbally using substitution and elimination.

In grade eight, students expand their understanding of geometric figures to understand similarity and congruency. They recognize and apply dilations, translations, rotations, and reflections to two-dimensional figures. Students use transformations to make arguments about similarity and congruency. They build on their knowledge of angles to understand relationships between interior and exterior angles in a triangle, as well as angles created when parallel lines are cut by a transversal.

In eighth grade, students gain an understanding of the relationship between the legs and hypotenuse of a right triangle and use it to derive and prove the Pythagorean theorem. They use the Pythagorean theorem to find unknown side lengths of right triangles and prove that triangles are right triangles. They build upon their knowledge of volume and circles to find the volume of cylinders, cones, and spheres, and use these formulas to solve problems.

Eighth graders extend their understanding to include bivariate data, recognizing the relationship between two quantities. They construct and interpret two-way tables and scatter plots and informally find a line of best fit to represent a relationship between variables.

Students use linear equations representing bivariate data to solve problems and interpret linear equations in the context of bivariate data.

Students taking grade 8 testlets have access to a calculator for selected testlets. They also have access to the Grade 8 Student Reference Sheet.

Strands

Strand 1: Understand and Use Irrational Numbers

Students identify whether a number is rational or irrational. They approximate or evaluate square and cube roots, locate these values on a number line, and order expressions involving square or cube roots from least to greatest. They use square and cube roots to find the radius or volume of cylinders, cones, and spheres. They write non-terminating decimals in fraction form.

Attributes

- The student approximates the value of irrational numbers and uses irrational numbers to solve problems.
- The student solves equations with square roots and cube roots.

CCSS-M

- 8.EE.A.2, 8.G.C.9, 8.NS.A.1, 8.NS.A.2

Additional Specifications

- Students may not use a calculator for this strand.

Strand 2: Exponent Rules and Scientific Notation

Students apply the properties of exponents to identify equivalent expressions. They use scientific notation to describe very large or very small quantities. They solve problems in real-world contexts by adding, subtracting, multiplying, or dividing numbers given in scientific notation.

Attributes

- The student applies properties of exponents to produce equivalent expressions.
- The student uses scientific notation to solve problems.

CCSS-M

- 8.EE.A.1, 8.EE.A.3, 8.EE.A.4

Additional Specifications

- Students may not use a calculator for this strand.

Strand 3: Understand Functions

Students identify whether a relation is a function and whether a function is linear or non-linear. They identify ordered pairs that are located on the graph of a function. They describe qualitatively the graph of a function.

Attributes

- The student recognizes a function represented in different ways and identifies points on the function.
- The student describes a functional relationship between two quantities.

CCSS-M

- 8.F.A.1, 8.F.B.5

Additional Specifications

- Functions are represented as equations, tables, graphs, and sets of ordered pairs.

Strand 4: Compare and Interpret Functions

Students identify and compare the slope, *y*-intercept, and *x*-intercept of functions. They interpret functional relationships, represented in different ways, as linear or non-linear in real-world contexts.

Attributes

- The student interprets the equations of linear functions.
- The student compares properties of functions represented in different ways.

CCSS-M

- 8.F.A.2, 8.F.A.3

Additional Specifications

- Functions are represented as equations, tables, graphs, and sets of ordered pairs.

Strand 5: Construct Functions

Students write an equation to model a function. They interpret a functional relationship, represented in different ways, in real-world contexts.

Attributes

- The student constructs a function.
- The student interprets and analyzes function components.

CCSS-M

- 8.F.B.4

Additional Specifications

- Functions are represented as equations, tables, graphs, and sets of ordered pairs.

Strand 6: Linear Equations in One Variable

Students identify whether an equation has exactly one, zero, or infinitely many solutions and justify their answer. They identify steps in solving an equation, describe errors in solution steps, and solve equations.

Attributes

- The student recognizes equations with different types of solutions.
- The student solves linear equations with rational number coefficients.

CCSS-M

- 8.EE.C.7, 8.EE.C.7.a, 8.EE.C.7.b

Additional Specifications

- All equations are linear.

Strand 7: Proportional Relationships and Lines

Students write an equation to represent a graph of a line. They compare the slopes of sections of the same line. They graph a line to model an equation or a real-world situation. They identify the slope and *y*-axis of a line and interpret these values in a mathematical or real-world context. They compare the slope and intercepts of two lines.

Attributes

- The student uses slope and the equation of a line to solve problems.
- The student graphs proportional relationships and compares them when represented in different ways.

CCSS-M

- 8.EE.B.5, 8.EE.B.6

Additional Specifications

- n/a

Strand 8: Systems of Equations

Students visually estimate the solution to a system of equations. They graph lines to represent the equations in a system and plot a point at the solution. They solve systems in mathematical contexts. They solve problems in real-world contexts that can be represented by a system of equations and interpret the solution.

Attributes

- The student solves a system of equations algebraically or from a verbal description.
- The student estimates the solution of a system of equations from a graph.

CCSS-M

- 8.EE.C.8, 8.EE.C.8.a, 8.EE.C.8.b, 8.EE.C.8.c

Additional Specifications

- Systems of equations can be solved algebraically with basic substitution or elimination.
- Students solve real-world and mathematical problems that lead to a system of linear equations.

Strand 9: Pythagorean Theorem

Students use the Pythagorean theorem to find an unknown side length in real-world contexts. They calculate the distance between points on a coordinate grid using the Pythagorean theorem and calculate the perimeter of polygons drawn on a coordinate grid. They use the Pythagorean theorem to determine if a triangle is a right triangle.

Attributes

- The student uses the Pythagorean theorem to find unknown side lengths.
- The student uses the Pythagorean theorem to find the distance between two points in a coordinate system.

CCSS-M

- 8.G.B.6, 8.G.B.7, 8.G.B.8

Additional Specifications

- When finding distance between two points, points should be represented on a coordinate grid.

Strand 10: Geometric Transformations

Students apply properties of translations, reflections, and rotations of geometric figures to describe attributes of an image or its preimage. They describe transformations that would map a preimage onto its image. They demonstrate that the preimage and image are congruent using geometric transformations. They identify the coordinates of vertices of the image based on the coordinates of the preimage and given transformations.

Attributes

- The student uses transformations to show the congruence of two-dimensional figures.
- The student describes the effects of transformations on two-dimensional figures.

CCSS-M

- 8.G.A.1, 8.G.A.2, 8.G.A.3

Additional Specifications

- n/a

Strand 11: Similarity and Congruence

Students apply properties of rigid transformations and dilations to determine whether two figures are congruent, similar, or neither. They identify and justify measures of angles based on angle relationships.

Attributes

- The student uses transformations to show the similarity of two-dimensional figures.
- The student uses informal arguments to show relationships between angles.

CCSS-M

- 8.G.A.4, 8.G.A.5

Additional Specifications

- All dilations are performed with respect to the origin on a coordinate grid.

Strand 12: Bivariate Data

Students interpret data presented in a scatter plot to identify associations between two variables in a real-world context. They interpret the equation of the line of best fit and visually assess whether a given line is the line of best fit. They analyze data in two-way association tables and determine relative frequencies.

Attributes

- The student uses a line of best fit to model data and solve problems.
- The student describes data patterns.

CCSS

- 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4

Additional Specifications

- n/a

Common Errors and Misconceptions

The following list gives examples of the common Grade 8 errors and misconceptions that distractors and anticipated incorrect responses are tagged with pre-administration:

- The student incorrectly applies properties of exponents (e.g., $x^5 \cdot x^2 = x^{10}$).
- The student assumes all functions must be linear.
- The student interchanges equations having no solution with equations having a solution of all real numbers.
- The student incorrectly finds slope as the ratio of change in *x* to change in *y*.
- The student incorrectly substitutes for the wrong variable in a system of equations.

- The student misidentifies the hypotenuse of a triangle when applying the Pythagorean theorem.

Appendix A – Real-World Context vs. Real-World Problems

Throughout the CCSS-M, standards call for students to "solve real-world problems." When developing assessments, it is important to differentiate between items that present a context to frame or scaffold a problem and items that require students to apply problem-solving skills.

Many items contain word problems, which we will define as contextual problems. These problems include language that may clarify, provide scaffolding, or help students visualize the mathematics they are being asked to do. For example, a student may be asked to solve the following word problem:

Prajit has 2 boxes. Each box has 3 markers. How many markers are there in all?

This problem provides a context for students to visualize the expression 2×3 and scaffolds the concept of multiplication. These are more likely to be low complexity items. Sometimes, these items may seem contrived.

A real-world problem differs in that the context itself is the problem and is higher in complexity, and usually requires modeling and the application of mathematics. These are problems that students may encounter in the real world. Students may be analyzing real-world data and forming conclusions. These problems should spark curiosity and involve critical thinking.

Appendix B – Addition and Subtraction Situations

Adapted from the *Progression for the Common Core State Standards for Mathematics*.

Add to

Unknown	Examples	Situation equations
Result	• There are 3 markers in a box. Jamie puts 5 more markers in the box. Now how many markers are in the box?	$3 + 5 = ?$

Change	• There are 3 markers in a box. Jamie puts some more markers in the box. Now there are 8 markers in the box. How many markers did Jamie put in?	$3 + ? = 8$
Start	• There are some markers in a box. Jamie puts 5 more markers in the box. Now there are 8 markers in the box. How many markers were in the box to begin with?	$? + 5 = 8$

Take from

Unknown	Examples	Situation equations
Result	• There are 8 markers in a box. Jamie takes 5 markers out of the box. Now how many markers are in the box?	$8 - 5 = ?$
Change	• There are 8 markers in a box. Jamie takes some markers out of the box. Now there are 3 markers in the box. How many markers did Jamie take out of the box?	$8 - ? = 3$
Start	• There are some markers in a box. Jamie takes 5 markers out of the box. Now there are 3 markers in the box. How many markers were in the box to begin with?	$? - 5 = 3$

Put together / take apart

Unknown	Examples	Situation equations
Total	• Jamie puts 3 markers and 5 crayons in a box. How many total markers and crayons are in the box?	$3 + 5 = ?$
One addend*	• Jamie puts 3 markers and some crayons in a box. There are a total of 8 markers and crayons in the box. How many crayons did Jamie put in the box?	$3 + ? = 8$ $8 - 3 = ?$
Both addends	• There are some markers and some crayons in a box. There are a total of 8 markers and crayons in the box. How many markers and how many crayons can be in the box?	$? + ? = 8$

*Either addend could be the unknown in this situation.

Compare

Unknown	Examples*	Situation equations
Difference	• Jamie has 3 markers. Alex has 8 markers. How many more markers does Alex have than Jamie? • Jamie has 3 markers. Alex has 8 markers. How many fewer markers does Jamie have than Alex?	$3 + ? = 8$ $8 - 3 = ?$
Greater quantity	• Alex has 5 more markers than Jamie. Jamie has 3 markers. How many markers does Alex have? • Jamie has 5 fewer markers than Alex. Jamie has 3 markers. How many markers does Alex have?	$3 + 5 = ?$
Lesser quantity	• Alex has 5 more markers than Jamie. Alex has 8 markers. How many markers does Jamie have? • Jamie has 5 fewer markers than Alex. Alex has 8 markers. How many markers does Jamie have?	$8 - 5 = ?$ $? + 5 = 8$

*Comparison situations can include either “fewer” or “more.”

Appendix C – Multiplication and Division Situations

Adapted from the *Progression for the Common Core State Standards for Mathematics*.

	Unknown Product	Group Size Unknown	Number of Groups Unknown
Equal Groups	• Jo has 4 bags with 5 brushes in each bag. How many brushes does Jo have in all?	• Jo puts 20 brushes equally into 4 bags. How many brushes are in each bag?	• Jo puts 20 brushes into bags, with 5 in each bag. How many bags does Jo put brushes in?
Arrays (can also be worded using columns, or using both rows and columns)	• Jo puts brushes into 4 rows with 5 in each row. How many brushes does Jo have?	• Jo puts 20 brushes into 4 equal rows. How many brushes are in each row?	• Jo puts 20 brushes into equal rows, with 5 in each row. How many rows are there?
Compare	• Jo has 4 bags of brushes. Riley has 5 times as many brushes. How many brushes does Riley have?	• Riley has 20 brushes, which is 5 times as many as Jo. How many	• Jo has 4 brushes and Riley has 20 brushes. How many times as many brushes

		brushes does Jo have?	does Riley have than Jo?
Area	• What is the area of a 4-inch by 6-inch rectangle?	• A rectangle is 4 inches long and has an area of 24 square inches. What is the other side length?	• A rectangle is 6 inches long and has an area of 24 square inches. What is the other side length?

Note: Measurement contexts can be used in each situation.

Appendix D – Student Reference Sheets

Grade 3

No reference sheet is available to students taking grade 3 testlets.

Grade 4

No reference sheet is available to students taking grade 4 testlets.

Grade 5

Equivalent Measurements

Length	Mass/Weight	Liquid Volume
1 mile = 5280 feet	1 pound = 16 ounces	1 cup = 8 fluid ounces
1 mile = 1760 yards	1 ton = 2000 pounds	1 pint = 2 cups
		1 quart = 2 pints
		1 gallon = 4 quarts
		1 liter = 1000 cubic centimeters

Formulas

Right Rectangular Prism: $V = B \times h$ or $V = l \times w \times h$

Grade 6

Equivalent Measurements

Length	Mass/Weight	Liquid Volume
1 inch = 2.54 centimeters	1 pound = 16 ounces	1 cup = 8 fluid ounces

1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers
1 kilometer = 0.62 mile

1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallons
1 liter = 1000 cubic centimeters

Formulas

Right Rectangular Prism: $V = Bh$ or $V = lwh$

Triangle: $A = \frac{1}{2}bh$

Grade 7

Equivalent Measurements

Length	Mass/Weight	Liquid Volume
1 inch = 2.54 centimeters	1 pound = 16 ounces	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 0.454 kilograms	1 pint = 2 cups
1 mile = 5280 feet	1 kilogram = 2.2 pounds	1 quart = 2 pints
1 mile = 1760 yards	1 ton = 2000 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers		1 gallon = 3.785 liters
1 kilometer = 0.62 mile		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

Formulas

Right Rectangular Prism: $V = Bh$ or $V = lwh$

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Circle: $A = \pi r^2$

Circle: $C = \pi d$ or $C = 2\pi r$

General Prisms: $V = Bh$

Grade 8

Equivalent Measurements

Length	Mass/Weight	Liquid Volume
1 inch = 2.54 centimeters	1 pound = 16 ounces	1 cup = 8 fluid ounces

1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers
1 kilometer = 0.62 mile

1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallons
1 liter = 1000 cubic centimeters

Formulas

Right Rectangular Prism: $V = Bh$ or $V = lwh$

Triangle: $A = \frac{1}{2}bh$

Parallelogram: $A = bh$

Circle: $A = \pi r^2$

Circle: $C = \pi d$ or $C = 2\pi r$

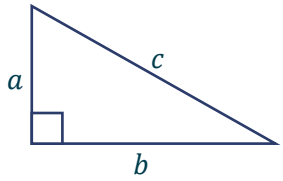
General Prisms: $V = Bh$

Cylinder: $V = \pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

Cone: $V = \frac{1}{3}\pi r^2 h$

Pythagorean Theorem: $a^2 + b^2 = c^2$



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